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Entanglement dynamics for a solid polariton system at zero and finite temperatures

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Abstract. The dynamics of the entanglement for a solid polariton system is investigated. The polariton system is a photon-phonon complex and its time-dependent characteristic function in the Wigner representation for the system is obtained analytically. It is found that when the photon field is initially prepared in the squeezed vacuum state, and the phonon in the thermal state, the polariton system can evolve into a two-mode Gaussian mixed state. The entanglement between photon and phonon turns out to be apparently dependent on the squeezing parameter and exhibits a critical behavior with respect to the temperature.

PACS. 03.67.Mn Entanglement production, characterization and manipulation – -71.36+c Polaritons (including photon-phonon and photon-magnon interactions)

1 Introduction

One of the main tasks of quantum information theory is to quantify the entanglement and the quantum correlations that quantum states possess. Recently a great deal of attention has been devoted to quantum information processing with canonical continuous variables, and various protocols for quantum communication and computation have been developed based on continuous variables [1]. Among all the quantum states in continuous variable systems, Gaussian states play a central role in the theory of entanglement for continuous variable systems due to the fact that experimentally they are relatively easy to create and arise naturally as states of the light field of laser [2] or in atomic ensembles interacting with light [3]. In spite of the fact that Gaussian states live in infinite-dimensional bosonic Fock space, all of their physical features are indeed captured by the so-called covariance matrix. From this finite-dimensional matrix one is able to extract straightforwardly various measures of entanglement such as entanglement of formation (EOF) for two-mode symmetric Gaussian states [4], lower bounds on the entanglement of asymmetric Gaussian states [5], negativity and logarithmic negativity (LN) [6,7]. This is the most popular one since it is comparatively easy to calculate for all two-mode Gaussian states. With this aim in mind it is often useful to cast the covariance matrix in standard form. This is possible via local unitary operations which do not change the entanglement of the state [8,9].

On the other hand, the study of the behavior of polariton systems has long been an active research area in condensed matter physics, especially in semiconductor optical microcavities. Polaritons are collective excitations of phonons or excitons of a crystal generated from a coherent linear interaction between a polar material mode and cavity field. The science of polaritons through semiconductor microcavities has made progress in experiments including squeezed polariton generation using polariton degenerate four-wave mixing [10] and probing of polariton quantum correlations [11]. A scheme for the generation of branch-entangled pairs of microcavity polaritons [12] has been proposed. Polariton condensation is also a subject of great interest [13]. The possibility of using a solid medium to store few-photon laser pulses has been investigated [14], where a theoretical analysis of storing quantum information in solids using a polariton formalism was carried out. Such a scheme would be well worth considering, as solids have a number of advantages over gases and are easier to prepare and store. Therefore, it is interesting to study entanglement in solid polariton systems. The polariton system we consider here is a photon-phonon complex which results when light falls on a solid-state material and interacts with the vibrating lattice [15,16]. It has been shown that the phonon and photon subsystem can exhibit interesting nonclassical behavior [17]. The purpose of this paper is to investigate the entanglement dynamics of the polariton system by using entanglement measures of continuous variables, namely the EOF [4] and the LN [6], respectively. These two measures are directly evaluated

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in accordance with the schemes using the Wigner characteristic function of the coupling system. We compute the time evolution of the entanglement, starting from an initial thermal state of the phonon for both zero and finite temperatures. We show that our system is exactly solvable and find that if the cavity field is initially prepared in a squeezed state and the phonon in a thermal state, the state of the coupling system can evolve into a two-mode Gaussian mixed state. The entanglement properties of the state which change with parameters of the system are examined in terms of the covariance matrix of the Gaussian state. The dynamical behavior of the entanglement between photon and phonon turns out to be apparently dependent on the squeezing parameter and exhibits a critical behavior with respect to the temperature.

2 A polariton system and its evolution

Polaritons are collective excitations of a crystal generated from a coherent linear interaction between a polar material mode and an electromagnetic field [18]. When light falls on a solid-state material and interacts with the vibrating lattice, a photon-phonon complex which results in a polariton is formed. Phonons arise as harmonic excitations due to the vibrational modes of the ions in the polariton system. From a quantum computing point of view the information is encoded in the phonon degrees of freedom while the radiation field acts as a medium for the interaction among the phonons. The purpose of this paper is to investigate the entanglement properties of the photon-phonon system. In order to perform some calculations, we need to specify a quasi-realistic model [19]. In many situations, a rotating wave approximation for the optical frequencies is appropriate. However, the truncation may cause the loss of some correlation contributing to the entanglement of the system. It has been shown that the non-rotating terms can enhance the nonclassical behavior of the system such as squeezing [21]. Therefore, the non-rotating wave term $a^\dagger b^\dagger + ab$ in the Hamiltonian is preserved. If only linear effects are taken into account, the Hamiltonian of the simplest possible model involving one mode of photon field interacting with a single optical phonon mode is given by [15]

$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + \kappa(a^\dagger b^\dagger + ab + a^\dagger b + b^\dagger a) \qquad (1)$$

where $a^\dagger(a)$ is the creation (annihilation) operator for the photon with frequency ω_a, $b^\dagger(b)$ denotes the creation (annihilation) operator for an optical phonon with phonon frequency ω_b, and κ is the phonon-photon coupling strength. Throughout we employ units with $\hbar = c = k_B = 1$. Physically, the first and second terms of Hamiltonian (1) represent the energy spectra of the free photon field and the free phonon field, respectively, and the third term describes the interaction between the two subsystems. In general, the frequency of the phonon and the coupling strength lie in the optical region. For example, the decay time of the transverse optical phonon polariton has been measured by impulsive stimulated Raman scattering [20] and the frequency of the phonon for GaP is in infra-red region $\sim 10^{13}$ Hz. For simplicity, we restrict ourselves to the resonant case, i.e., $\omega_a = \omega_b = \omega$. Using the following unitary transformation of the bosonic modes

$$\begin{aligned} \alpha &= A_+ a - A_- a^\dagger + A_+ b - A_- b^\dagger, \\ \beta &= -B_+ a + B_- a^\dagger + B_+ b - B_- b^\dagger \end{aligned} \qquad (2)$$

and choosing suitable parameters

$$\begin{aligned} A_\pm &= \frac{1}{2\sqrt{2}} \left[\left(\frac{\omega}{\omega - 2\kappa} \right)^{1/4} \pm \left(\frac{\omega - 2\kappa}{\omega} \right)^{1/4} \right], \\ B_\pm &= -\frac{1}{2\sqrt{2}} \left[\left(\frac{\omega}{\omega + 2\kappa} \right)^{1/4} \pm \left(\frac{\omega + 2\kappa}{\omega} \right)^{1/4} \right], \end{aligned} \qquad (3)$$

the Hamiltonian can be diagonalised as

$$H = E_\alpha \alpha^\dagger \alpha + E_\beta \beta^\dagger \beta + E_0 \qquad (4)$$

where the operators α and β are new Bose operators in the polariton system and satisfy the usual commutation relations $[\alpha, \beta] = 0$, $[\alpha, \alpha^\dagger] = 1$ and $[\beta, \beta^\dagger] = 1$. The indices α and β specify the two branches of the energy spectrum, where the energy E_α and E_β are given by $E_\alpha = \sqrt{\omega^2 - 2\kappa\omega}$ and $E_\beta = \sqrt{\omega^2 + 2\kappa\omega}$ respectively, and E_0 is the ground state energy of the system. The diagonalized Hamiltonian (4) means that one has "dressed" the phonons, and there is no entanglement between the two polariton branches for an initial separable state of polariton. However, the interaction between photon and phonon produces the coherence of their subsystems that is necessary for entanglement. For convenience, we express the transformation relation equation (2) in matrix form, which reads

$$\left(\alpha, \alpha^\dagger, \beta, \beta^\dagger\right)^T = S \left(a, b^\dagger, a, b^\dagger\right)^T \qquad (5)$$

where T represents the transpose of a matrix and

$$S = \begin{pmatrix} A_+ & -A_- & A_+ & -A_- \\ -A_- & A_+ & -A_- & A_+ \\ -B_+ & B_- & B_+ & -B_- \\ B_- & -B_+ & -B_- & B_+ \end{pmatrix} \qquad (6)$$

is a real 4×4 transformation matrix. The time dependent operators $\alpha(t)$ and $\beta(t)$ can be easily obtained through solving their Heisenberg equation of motion, and is expressed as

$$\begin{aligned} &\left(\alpha(t), \alpha^\dagger(t), \beta(t), \beta^\dagger(t)\right)^T = \\ &\qquad O(t) \left(\alpha(0), \alpha^\dagger(0), \beta(0), \beta^\dagger(0)\right)^T \end{aligned} \qquad (7)$$

where $O(t)$ is given by

$$O(t) = \begin{pmatrix} \exp(-iE_\alpha t) & 0 & 0 & 0 \\ 0 & \exp(iE_\alpha t) & 0 & 0 \\ 0 & 0 & \exp(-iE_\beta t) & 0 \\ 0 & 0 & 0 & \exp(iE_\beta t) \end{pmatrix}. \qquad (8)$$

Together with the transformation relations equations (5) and (7), we have

$$(a(t), a^\dagger(t), b(t), b^\dagger(t))^T = V(t)(a(0), a^\dagger(0), b(0), b^\dagger(0))^T \quad (9)$$

where $V(t) = S^{-1}O(t)S$.

In order to obtain the time-dependent characteristic function in the Wigner representation for the polariton system, it is advantageous to make use of the position and momentum operators

$$q_a = \frac{1}{\sqrt{2}}(a^\dagger + a), \quad p_a = i\frac{1}{\sqrt{2}}(a^\dagger - a)$$
$$q_b = \frac{1}{\sqrt{2}}(b^\dagger + b), \quad p_b = i\frac{1}{\sqrt{2}}(b^\dagger - b) \quad (10)$$

to rewrite the relation between the boson operators and the quadrature operators in the form

$$(q_a, p_a, q_b, p_b)^T = Q(a, a^\dagger, b, b^\dagger)^T \quad (11)$$

where $Q = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ -i & i & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -i & i \end{pmatrix}$.

Hence, the evolution of $(q_a(t), p_a(t), q_b(t), p_b(t))^T$ can be obtained from equations (9) and (10) and reads

$$(q_a(t), p_a(t), q_b(t), p_b(t))^T = U(t)(q_a, p_a, q_b, p_b)^T \quad (12)$$

where $U(t) = QV(t)Q^{-1}$.

The Gaussian state is one whose Wigner characteristic function, defined by [22]

$$\chi(\xi_a, \xi_b) = Tr[\rho \exp(-\xi_a^* a + \xi_a a^\dagger - \xi_b^* b + \xi_b b^\dagger)] \quad (13)$$

is a Gaussian. The Wigner characteristic function of a Gaussian system can be written in the following compact form

$$\chi(\xi_a, \xi_b) = \exp\left[-\frac{1}{2}\Lambda^T M \Lambda\right] \quad (14)$$

with $\Lambda = [\mathrm{Im}(\xi_a), \mathrm{Re}(\xi_a), \mathrm{Im}(\xi_b), \mathrm{Re}(\xi_b)]^T$, where Re(Im) denotes the real (imaginary) part of a function, and M is the so-called covariance matrix [8]. The characteristic function can also be written as

$$\chi(\xi_a, \xi_b) = Tr\left\{\rho \exp\left[i\sqrt{2}\Lambda^T (q_a, p_a, q_b, p_b)^T\right]\right\} \quad (15)$$

and its time evolution can thus be expressed as

$$\chi(\xi_a, \xi_b, t) = Tr\{\rho \exp\left\{i\sqrt{2}\left[U^T(t)\Lambda\right]^T (q_a, p_a, q_b, p_b)^T\right\}. \quad (16)$$

It can be seen that the time-dependent characteristic function $\chi(\xi_a, \xi_b, t)$ can simply be obtained from $\chi(\xi_a, \xi_b)$ by the substitution $\Lambda \rightarrow U^T(t)\Lambda$, and is given by

$$\chi(\xi_a, \xi_b, t) = \exp\left\{-\frac{1}{2}[U^T(t)\Lambda]^T M U^T(t)\Lambda\right\} \quad (17)$$

which directly yields the covariance matrix

$$M(t) = U(t) M U^T(t). \quad (18)$$

We assume that the photon field is initially in the squeezed vacuum state

$$\rho_a = S(r)\left|0\right\rangle_{a\,a}\left\langle 0\right| S^\dagger(r) \quad (19)$$

where $S(r) = \exp\left\{r\left[a^2 - (a^+)^2\right]/2\right\}$ is a squeezed operator [23] with the real squeezing parameter r which can be physically understood in the form $\langle a^\dagger a\rangle = \sinh^2(r)$, and the phonon field is initially prepared in the thermal state

$$\rho_b = \frac{1}{1+\bar{n}}\sum_{n=0}^{\infty}\left(\frac{\bar{n}}{1+\bar{n}}\right)^n |n\rangle_{b\,b}\langle n| \quad (20)$$

where $|n\rangle_b$ is the phonon number state and $\bar{n} = 1/[\exp(\omega/T) - 1]$ is the thermal average phonon number for thermal equilibrium at a certain temperature. In the case of zero temperature, the phonon is prepared in its vacuum state $|0\rangle_b$, which allows us to mainly investigate the influence of an initial squeezed field on the entanglement dynamics of the polariton system. At finite temperature, the density matrix of the system is the summation of all states with their Boltzmann weights. Therefore, for very high temperature the density matrix consists of an almost uniform distribution in the state space, which leads to the distribution of the number of the phonon being chaotic and a vanishing of entanglement. The covariance matrix of the initial state of system $M(t=0)$ can be obtained as

$$M = \begin{pmatrix} \cosh(2r)-\sinh(2r) & 0 & 0 & 0 \\ 0 & \cosh(2r)+\sinh(2r) & 0 & 0 \\ 0 & 0 & 2\bar{n}+1 & 0 \\ 0 & 0 & 0 & 2\bar{n}+1 \end{pmatrix}. \quad (21)$$

The time-dependent covariance matrix of the polariton system can then be obtained from equations (18) and (21) as follows

$$M(t) = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix} \quad (22)$$

with 2×2 matrices

$$A = \begin{pmatrix} f_1^- - 2\mathrm{Re}(f_2^-) & -2\mathrm{Im}(f_2^-) \\ -2\mathrm{Im}(f_2^-) & f_1^- + 2\mathrm{Re}(f_2^-) \end{pmatrix},$$
$$B = \begin{pmatrix} f_1^+ - 2\mathrm{Re}(f_2^+) & -2\mathrm{Im}(f_2^+) \\ -2\mathrm{Im}(f_2^+) & f_1^+ + 2\mathrm{Re}(f_2^+) \end{pmatrix}, \quad (23)$$
$$C = \begin{pmatrix} \mathrm{Re}(f_4 - f_3) & \mathrm{Im}(f_3 - f_4) \\ \mathrm{Im}(f_3 + f_4) & \mathrm{Re}(f_4 + f_3) \end{pmatrix}.$$

The explicit expression for f_i in $M(t)$ is given in the Appendix. States with a Gaussian Wigner distribution appear naturally in every quantum system which can be described or approximated by a quadratic bosonic Hamiltonian. The evolved state of the coupled system is a Gaussian one in the sense that if its Wigner characteristic function is a Gaussian function of the continuous variables. The entanglement between two subsystems that are described in terms of continuous variables has been established [4,6], and the continuous variable entanglement between the vibrational modes of two coupled oscillators has been recently studied [24] using a general linear mode transformation. From equation (17), one can see that the time dependent characteristic function $\chi(\xi_a, \xi_b, t)$ is explicitly a Gaussian-style function, which means that an initial Gaussian state $\rho(0) = \rho_a \otimes \rho_b$ of the polariton system will maintain its Gaussian characteristic during the time evolution, which is consistent with the fact that an initial Gaussian state will be Gaussian at all times under the evolution of the Hamiltonians (1). In what follows, we will see that entanglement properties as a function of time are determined by the two-mode Gaussian wave packets for both zero and finite temperature.

3 Continuous variable entanglement

We now discuss the time behavior of the entanglement of the polariton system and explore the influence of the squeezing parameter and the temperature on the entanglement. It is well known that for any two-mode covariance matrix M there exists a local symplectic operation which takes M to the so-called standard form M_s [8,9]

$$M_s = \begin{pmatrix} n & 0 & k_x & 0 \\ 0 & n & 0 & -k_p \\ k_x & 0 & m & 0 \\ 0 & -k_p & 0 & m \end{pmatrix} \tag{24}$$

where $k_x \geqslant k_p \geqslant 0$. Since the transformation that gives M_s is local, it does not affect the entanglement between the two subsystems. The correlations n, m, k_x and k_p are determined by the four local symplectic invariants

$$\begin{aligned} &\mathrm{Det}\, A = n^2, \mathrm{Det}\, B = m^2, |\mathrm{Det}\, C| = k_x k_p, \\ &\mathrm{Det}\, M = (mn - k_x^2)(mn - k_p^2). \end{aligned} \tag{25}$$

Therefore, the standard form corresponding to any covariance matrix is unique. States whose standard form fulfills $m = n$ are said to be symmetric. Separability criteria for a two-mode Gaussian state have been established [8,9]. The covariance matrix can be used to derive an analytical expression of entanglement for the polariton system. In order to give a quantitative description of how the entanglement of the Gaussian state changes we consider the following two cases. We start from different initial preparations i.e., zero and finite temperatures, and adopt either the EOF [4] or the LN [6] approach according to whether their Gaussian states are in symmetric or asymmetric form. These calculations involve covariance matrices both in standard and non-standard form.

3.1 Zero temperature situation

Now we consider the zero temperature case and suppose that the photon field is initially in the squeezed vacuum state $S(r)\left|0\right\rangle_a$ and the phonon field is in the vacuum state $\left|0\right\rangle_b$. As time evolves we get quantum entanglement between the two subsystems due to their interaction. There is a particular manifestation of nonclassical photon statistics for the squeezed vacuum state, which can be generated using a non-degenerate optical parametric amplifier [24]. Recently, there have been some proposals connecting the squeezing parameter with the entanglement of atomic states [25]. Our purpose here is to study what effect squeezing of the radiation field has on the entanglement of the polariton system. From equation (25), we can directly calculate the standard form of the covariance matrix M_s, whose matrix elements are determined to be

$$m = n = \sqrt{(f_1^-)^2 - 4\left|f_2^-\right|^2} = \sqrt{(f_1^+)^2 - 4\left|f_2^+\right|^2}, \tag{26}$$

$$k_x = k_p = \sqrt{|f_3|^2 - |f_4|^2}, \tag{27}$$

where f_i are given by the equation in the Appendix with $\bar{n}$ set to zero.

Remarkably, the polariton system evolves into a symmetric two-mode Gaussian state (invariant under interchange of the subsystems). Therefore we can directly evaluate the EOF of the system from the covariance matrix to quantify the entanglement of the system. The EOF is obtained by Giedke et al. [4] based on the sufficient and necessary condition for separability of the Gaussian system [8]. It is noteworthy that the EOF is a proper measure of the entanglement between two subsystems and is equal to the von Neumann entropy if the compound system remains in a pure state. However, unlike the von Neumann entropy, the EOF still works for mixed states. The EOF can be calculated by [4]

$$E_F(\Delta) = c_+(\Delta)\ln[c_+(\Delta)] - c_-(\Delta)\ln[c_-(\Delta)] \tag{28}$$

where $c_\pm(\Delta) = [(\Delta)^{-1/2} \pm (\Delta)^{1/2}]^2/4$, and $\Delta = \min(1, |n - k_x|)$. The state is entangled only when $\Delta < 1$.

To gain a better understanding of the entanglement evolution in the time domain, the EOF as a function of squeezing parameter r and scaled time ωt is plotted for $\kappa = 0.3\omega$ and $\omega = 10$ in Figure 1. It can be seen that at any given time, as the squeezing parameter of the cavity field increases, the entanglement increases monotonically. For a given squeezing parameter, the entanglement exhibits periodic behavior with time. The EOF oscillates with the period given by $\omega t \sim 4.99$ and shows the double peak structure within one main period. It reaches its maximum at the time points $\omega t \sim 1.27n$ with $n = 1, 3, 5, ...$ There is a useful connection between nonclassicality and inseparability. The larger the squeezing degree of the initial field, the stronger the entanglement at a certain time. The change of the squeezing degree has an apparent effect on the entanglement. In other words, the nonclassical property of the initial photon field contributes to the entanglement generation in the polariton system. In fact,

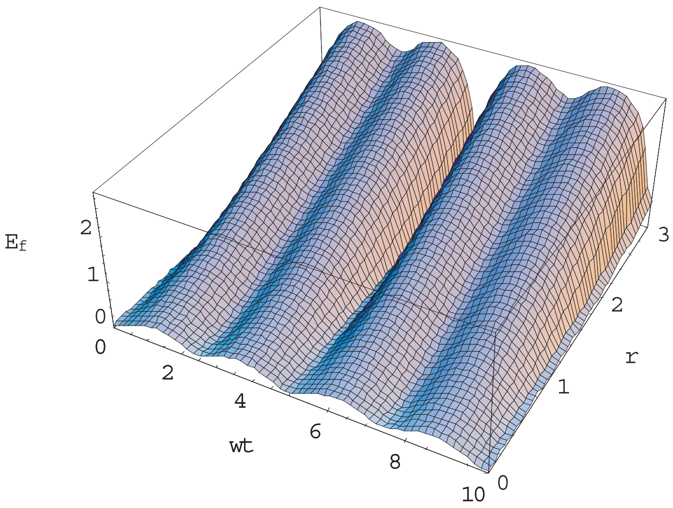

Fig. 1. EOF as a function of squeezing parameter r and scaled time ωt for $\kappa = 0.3\omega$ and $\omega = 10$. one can see EOF increases monotonically with increasing squeezing parameter for a given time.

Kraus et al. [26] have previously proved that in order to create as much entanglement as possible it is more efficient to squeeze the state locally first before commencing the interaction.

3.2 Finite temperature situation

We now focus on the case of finite temperature in which the phonon modes are in the thermal state given by equation (20), and investigate the influence of the temperature on the entanglement evolution. The results will be compared with the zero temperature case. Suppose the interaction between the phonons and the photons begins at $t = 0$. Initially the phonons are in the thermal state where the density matrix is given by equation (20) and the photons are in the squeezed state. We will see in the following that the state of the polariton system can evolve into an asymmetric Gaussian state where the EOF fails to quantify the entanglement. Therefore we use the LN [6], which is the most popular solution due to the fact that it is comparatively easy to calculate for all two mode Gaussian states. In the special instance of the symmetric two-mode Gaussian state, the EOF provides the same characterization of entanglement and is equivalent to the LN. It has been shown [27] that for any given covariance matrix the entanglement is lower bounded by that of a Gaussian state if it is measured in an appropriate way. When applying LN to a Gaussian approximation of a non-Gaussian state it can lead to an overestimation of the entanglement.

The LN is defined as $E_N(\rho) = \log_2 ||\rho^T||_1$, where ρ^T is the partial transpose of the density matrix ρ of the system, and $||\rho^T||_1 \equiv tr\left|\rho^T\right|$ denotes the trace norm i.e., the sum of the absolute values of ρ^T [28]. The concept of the negativity is based on the fact that a non-entangled state has necessarily a positive partial transpose according to the well-known peres-Horodecki criterion [29]. For all bipartite Gaussian states, fortunately, a positive partial transpose is also a sufficient condition [9]. The negativity essentially measures the degree to which ρ^T fails to be positive. Moreover, the LN determines upper bounds on the teleportation capacity and the entanglement of distillation [6]. The trace norm of the partially transposed density matrix can be computed from the so-called symplectic eigenvalues of the partial transpose of M. The symplectic eigenvalues encode essential information about the Gaussian state and provide powerful, simple ways to express its fundamental properties. The symplectic eigenvalues of the partial transposed matrix of the covariance matrix M can be found in terms of the following characteristic equation [6]

$$\lambda^4 - (\det A + \det B - 2\det C)\lambda^2 + \det M = 0 \tag{29}$$

and the two non-negative values are given by

$$\lambda_\pm = \frac{1}{\sqrt{2}}\{g_1 - 2g_2 + g_3 \pm [(g_1 - 2g_2 + g_3)^2 -4(g_2^2 + g_1g_3 + |f_3||f_4|g_1'g_3' + g_4(g_2')^2)]^{\frac{1}{2}}\}^{\frac{1}{2}} \tag{30}$$

where

$$g_1 = \frac{1}{4}(f_1^-)^2 - |f_2^-|^2, \quad g_1' = \frac{1}{2}f_1^- - |f_2^-|,$$
$$g_2 = \frac{1}{4}(|f_4|^2 - |f_3|^2), \quad g_2' = \frac{1}{2}(|f_4| - |f_3|),$$
$$g_3 = \frac{1}{4}(f_1^+)^2 - |f_2^+|^2, \quad g_3' = \frac{1}{2}f_1^+ - |f_2^+|,$$
$$g_4 = \frac{1}{2}(f_1^- f_1^+ + |f_2^-||f_2^+|). \tag{31}$$

It can easily be verified that the symplectic eigenvalue always satisfies $\lambda_+ > 1$ at any parameter condition and is not important for establishing the nonseparability of the state [6]. The symplectic eigenvalue λ_- can satisfy $\lambda_- < 1$ no matter what values the parameters take for the system and thus it is important to determine the nonseparability of the state. Since the state is Gaussian, it is completely characterized by the covariance matrix, and hence its degree of entanglement can be quantified by means of the LN [6]

$$E_N = \max\left[-\log_2 \lambda_-, 0\right]. \tag{32}$$

It is easy to see that E_N is a decreasing function of the smallest partially transposed symplectic eigenvalue λ_-. The symplectic eigenvalue completely quantifies the quantum entanglement of the two-mode Gaussian state. Furthermore, it turns out that the EOF [4] is also a monotonically deceasing function of λ_-, thus providing a quantification of the entanglement of symmetric states equivalent to the one provided by the negativities. The above expression of the entanglement given in equation (32) is not accessible by experiment. Fortunately, an experimentally reliable estimate [30] of continuous variable entanglement of a two-mode Gaussian state has been proposed. It is shown that the entanglement of states can be related

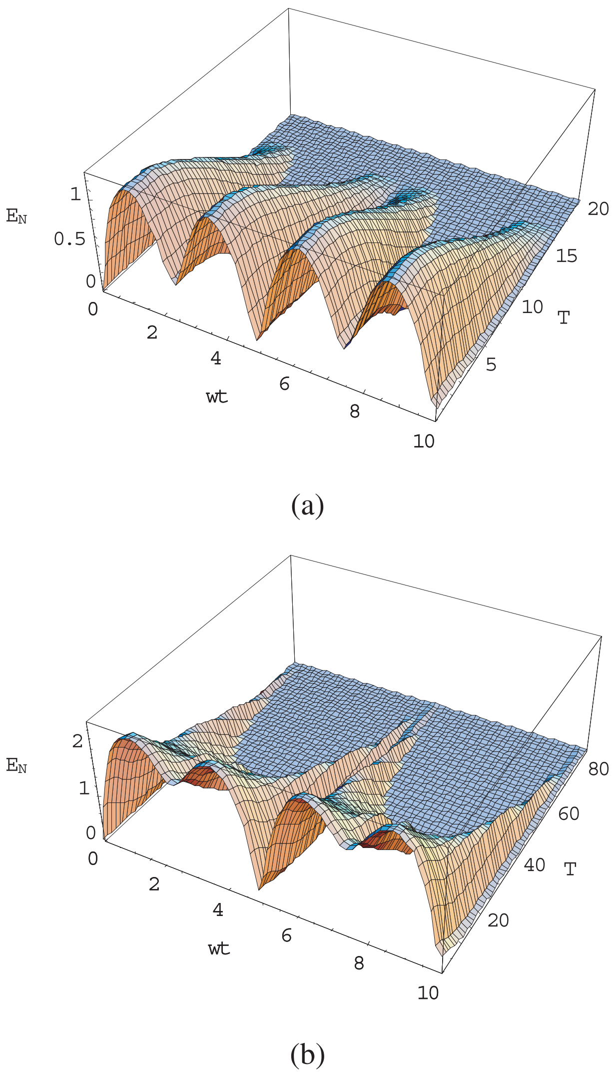

Fig. 2. The LN as a function of temperature T and scaled time ωt for $\kappa = 0.3\omega$, $\omega = 10$ and (a) $r = 0.5$; (b) $r = 1.5$.

to total and partial purities of the state, which can be measured even without homodyning [31].

In Figure 2, the LN as a function of the temperature T and the scaled time ωt are plotted with $k = 0.3\omega$, $\omega = 10$ for two different squeezing parameters, i.e., (a) $r = 0.5$; (b) $r = 1.5$.

The thermal phonon field is a highly chaotic field about which we have minimal information. The two subsystems are entangled due to their interaction. The entanglement can reduce the photon system to a mixed state when the phonon system variables are traced over. So we know that the mutual information between the two subsystems should become non-zero. It is shown in Figure 2 that the LN E_N decreases monotonically with increasing temperature T until it reaches a threshold value at which the entanglement vanishes. For different evolution times there exist different threshold temperatures.

In order to investigate explicitly the effect of the temperature on the entanglement, we consider the LN of the evolved system at a fixed time. One can see that the more the temperature rises, the more the entanglement decreases. In fact, the thermal fluctuations at high temperature always suppress the entanglement of the coupling system, namely, the LN is a decreasing function of the temperature. It is natural to expect that there exists a threshold temperature at which the LN becomes zero. By comparing Figure 2a with Figure 2b, one can see that the entanglement depends apparently on the squeezing degree. For a fixed temperature, similar to the zero temperature case, the entanglement increases with the increase of the squeezing parameter r. The threshold temperature is sensitive to the squeezing parameter r and converges quickly as it decreases. The larger the squeezing parameter r of the photon field, the higher the threshold temperature. This variation of the entanglement with temperature can be also seen for other times.

4 Conclusion and discussion

In conclusion, we have investigated the dynamics of entanglement in a polariton system with the phonon initially prepared in a thermal state and a photon field in a squeezed vacuum state. The time-dependent characteristic function for the system is solved analytically. It is found that the system evolves into a two-mode Gaussian mixed state, in particular a symmetric one when the temperature of the initial phonon state takes a zero value. The dynamical behavior of the entanglement between photon and phonon depends apparently on the squeezing degree r of the initial photon state and the temperature. The entanglement increases with increasing squeezing parameter at a fixed temperature, in other words, the nonclassical property of the initial photon field contributes to the entanglement generation. The entanglement at a given time shows monotonic behavior with respect to the temperature and vanishes when the temperature increases beyond the threshold value. In addition, the threshold value increases with the increase of the squeezing degree r.

Entanglement plays a key role in quantum information processing. However, due to decoherence, it is very difficult to generate and maintain an entangled pure state suitable for efficient quantum information processing. Practically, when the dissipation of photon and phonon modes is included, decoherence actually constitutes a serious impediment to producing a quantum computer. A long photon lifetime (1 ms) within a cavity allows for coherent dynamics lasting many Rabi floppings [32]. During the waiting time of a few Rabi floppings, photon losses could spoil the CV entangled state. The experimental value for the probability of losing a photon can be small during a waiting time of only several Rabi floppings. A more realistic model including the dissipation of the photon and phonon field is of great interest though the computationally modest goal will be extremely challenging experimentally [33].

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Appendix

$$
\begin{aligned}
f_1^{\pm} &= m^{+}(p_1^2 + p_3^2 - h_1^{+}) + 2\sinh(2r)h_7^{+} \\
&\quad \pm \frac{1}{2}\{m^{-}[4h_8 + \cos(E_\alpha t)\cos(E_\beta t)] - 4\sinh(2r)h_9\}, \\
f_2^{\pm} &= -m^{+}h_7^{+} + \frac{1}{2}\sinh(2r)(h_2^{+} - p_2^2 - p_4^2) \\
&\quad + \frac{1}{4}i[-m^{+}h_3^{+} - 2\sinh(2r)h_4^{+}] \\
&\quad \pm \frac{1}{4}\{-2m^{-}[2h_9 + 4h_8 - \cos(E_\alpha t)\cos(E_\beta t)] \\
&\quad - i[m^{-}h_5^{+} + 2\sinh(2r)h_6^{+}]\}, \\
f_3 &= -2m^{+}h_7^{-} + \sinh(2r)(p_4^2 - p_2^2 + h_2^{-}) \\
&\quad - \frac{1}{2}i[m^{+}h_3^{-} + \sinh(2r)h_4^{-}], \\
f_4 &= m^{+}(p_1^2 - p_3^2 - h_1^{-}) + 2\sinh(2r)h_7^{-} \\
&\quad - i[m^{-}h_6^{-} - \sinh(2r)h_7^{-}], \\
h_1^{\pm} &= p_2^2\cos(2E_\alpha t) \pm p_4^2\cos(2E_\beta t), \\
h_2^{\pm} &= p_1^2\cos(2E_\alpha t) \pm p_3^2\cos(2E_\beta t), \\
h_3^{\pm} &= p_2\sin(2E_\alpha t) \pm p_4\sin(2E_\beta t), \\
h_4^{\pm} &= p_1\sin(2E_\alpha t) \pm p_3\sin(2E_\beta t), \\
h_5^{\pm} &= p_2\sin(E_\alpha t)\cos(E_\beta t) \pm p_4\cos(E_\alpha t)\sin E_\beta t), \\
h_6^{\pm} &= p_1\sin(E_\alpha t)\cos(E_\beta t) \pm p_3\cos(E_\alpha t)\sin(E_\beta t), \\
h_7^{\pm} &= p_1p_2\sin^2(E_\alpha t) \pm p_3p_4\sin^2(E_\beta t), \\
h_8 &= (p_1p_3 + p_2p_4)\sin(E_\alpha t)\sin(E_\beta t), \\
h_9 &= (p_2p_3 + p_1p_4)\sin(E_\alpha t)\sin(E_\beta t), \\
m^{\pm} &= 2\bar{n} + 1 \pm \cosh(2r), \quad p_1 = A_+^2 + A_-^2, \\
p_2 &= 2A_+A_-, \quad p_3 = B_+^2 + B_-^2, \quad p_4 = 2B_+B_-
\end{aligned}
$$

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